

Inverse

Exercise 2.13

a. $A = \begin{pmatrix} -1 & i \\ 1+i & 0 \end{pmatrix}$

b. $B = \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$

c. $C = \begin{pmatrix} 1 & -1 & i \\ 1+i & 0 & \end{pmatrix}$

Solution Exercise 2.13

a.

$$A = \begin{pmatrix} -1 & i \\ 1+i & 0 \end{pmatrix}$$

$$\left(\begin{array}{cc|cc} -1 & i & 1 & 0 \\ 1+i & 0 & 0 & 1 \end{array} \right) \xrightarrow[\sim]{\frac{1}{2}-\frac{1}{2}iR_2 \leftrightarrow R_1} \left(\begin{array}{cc|cc} 1 & 0 & 0 & \frac{1}{2}-\frac{1}{2}i \\ -1 & i & 1 & 0 \end{array} \right) \xrightarrow[\sim]{1R_1 \rightarrow R_2} \left(\begin{array}{cc|cc} 1 & 0 & 0 & \frac{1}{2}-\frac{1}{2}i \\ 0 & i & 1 & \frac{1}{2}-\frac{1}{2}i \end{array} \right) \xrightarrow[\sim]{-1iR_2}$$

$$\left(\begin{array}{cc|cc} 1 & 0 & 0 & \frac{1}{2}-\frac{1}{2}i \\ 0 & 1 & -1 & i-\frac{1}{2}-\frac{1}{2}i \end{array} \right)$$

$$\text{inv } A = \begin{pmatrix} 0 & \frac{1}{2}-\frac{1}{2}i \\ -1 & i-\frac{1}{2}-\frac{1}{2}i \end{pmatrix}$$

b.

$$B = \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$$

$$\left(\begin{array}{cc|cc} 1 & i & 1 & 0 \\ i & 1 & 0 & 1 \end{array} \right) \xrightarrow[\sim]{-1iR_2 \leftrightarrow R_1} \left(\begin{array}{cc|cc} 1 & -1 & i & 0 \\ 1 & i & 1 & 0 \end{array} \right) \xrightarrow[\sim]{-1R_1 \rightarrow R_2} \left(\begin{array}{cc|cc} 1 & -1 & i & 0 \\ 0 & 2i & 1 & i \end{array} \right) \xrightarrow[\sim]{-\frac{1}{2}iR_2} \left(\begin{array}{cc|cc} 1 & -1 & i & 0 \\ 0 & 1 & -\frac{1}{2}i & \frac{1}{2} \end{array} \right) \xrightarrow[\sim]{iR_2 \rightarrow R_1}$$

$$\left(\begin{array}{cc|cc} 1 & 0 & \frac{1}{2} & -\frac{1}{2}i \\ 0 & 1 & -\frac{1}{2}i & \frac{1}{2} \end{array} \right)$$

$$\text{inv } B = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2}i \\ -\frac{1}{2}i & \frac{1}{2} \end{pmatrix}$$

c.

$$C = \begin{pmatrix} 1 & -1 & i \\ 1+i & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 & i & | & 1 & 0 \\ 1+i & 0 & 0 & | & 0 & 1 \end{pmatrix} \xrightarrow[\sim]{\frac{1}{2}-\frac{1}{2}iR_2 \leftrightarrow R_1} \begin{pmatrix} 1 & 0 & 0 & | & \frac{1}{2} & -\frac{1}{2}i \\ 1 & -1 & i & | & 1 & 0 \end{pmatrix} \xrightarrow[\sim]{-1R_1 \rightarrow R_2} \begin{pmatrix} 1 & 0 & 0 & | & \frac{1}{2} & -\frac{1}{2}i \\ 0 & -1 & i & | & 1 & -\frac{1}{2} + \frac{1}{2}i \end{pmatrix} \xrightarrow[\sim]{iR_2} \begin{pmatrix} 1 & 0 & 0 & | & \frac{1}{2} & -\frac{1}{2}i \\ 0 & 1 & i & | & -\frac{1}{2} & -\frac{1}{2}i \end{pmatrix}$$

$$\text{inv } C = \begin{pmatrix} 0 & \frac{1}{2} - \frac{1}{2}i \\ i & -\frac{1}{2} - \frac{1}{2}i \end{pmatrix}$$

Exercise 2.14

a. $A = \begin{pmatrix} -1 & -1 & -3 \\ 1 & 2 & 2 \\ 0 & 5 & -4 \end{pmatrix}$

b. $B = \begin{pmatrix} 1 & 3 & 3 \\ -1 & -4 & -4 \\ -1 & 3 & 2 \end{pmatrix}$

c. $C = \begin{pmatrix} -1 & -3 & -4 \\ 1 & 4 & -3 \\ 1 & 4 & -4 \end{pmatrix}$

d. $D = \begin{pmatrix} 0 & 1 & -4 \\ -1 & -1 & 1 \\ -2 & -1 & -1 \end{pmatrix}$

e. $E = \begin{pmatrix} 1 & 1 & 1 \\ -3 & -4 & -1 \\ 2 & 4 & -3 \end{pmatrix}$

f. $F = \begin{pmatrix} 1 & -2 & -2 \\ 3 & -5 & -5 \\ 1 & 0 & 1 \end{pmatrix}$

Solution Exercise 2.14

a.

$$A = \begin{pmatrix} -1 & -1 & -3 \\ 1 & 2 & 2 \\ 0 & 5 & -4 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} -1 & -1 & -3 & 1 & 0 & 0 \\ 1 & 2 & 2 & 0 & 1 & 0 \\ 0 & 5 & -4 & 0 & 0 & 1 \end{array} \right) \stackrel{-1 R_1}{\sim} \left(\begin{array}{ccc|ccc} 1 & 1 & 3 & -1 & 0 & 0 \\ 1 & 2 & 2 & 0 & 1 & 0 \\ 0 & 5 & -4 & 0 & 0 & 1 \end{array} \right) \stackrel{-1 R_1 \rightarrow R_2}{\sim} \left(\begin{array}{ccc|ccc} 1 & 1 & 3 & -1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 1 & 0 \\ 0 & 5 & -4 & 0 & 0 & 1 \end{array} \right) \stackrel{\frac{1}{5} R_3 \leftrightarrow R_2}{\sim}$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 3 & -1 & 0 & 0 \\ 0 & 1 & -\frac{4}{5} & 0 & 0 & \frac{1}{5} \\ 0 & 1 & -1 & 1 & 1 & 0 \end{array} \right) \stackrel{-1 R_2 \rightarrow R_3}{\sim} \left(\begin{array}{ccc|ccc} 1 & 1 & 3 & -1 & 0 & 0 \\ 0 & 1 & -\frac{4}{5} & 0 & 0 & \frac{1}{5} \\ 0 & 0 & -\frac{1}{5} & 1 & 1 & -\frac{1}{5} \end{array} \right) \stackrel{-1 R_2 \rightarrow R_1}{\sim} \left(\begin{array}{ccc|ccc} 1 & 0 & 3\frac{4}{5} & -1 & 0 & -\frac{1}{5} \\ 0 & 1 & -\frac{4}{5} & 0 & 0 & \frac{1}{5} \\ 0 & 0 & -\frac{1}{5} & 1 & 1 & -\frac{1}{5} \end{array} \right) \stackrel{-5 R_3}{\sim}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 3\frac{4}{5} & -1 & 0 & -\frac{1}{5} \\ 0 & 1 & -\frac{4}{5} & 0 & 0 & \frac{1}{5} \\ 0 & 0 & 1 & -5 & -5 & 1 \end{array} \right) \stackrel{-3\frac{4}{5} R_3 \rightarrow R_1}{\sim} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 18 & 19 & -4 \\ 0 & 1 & -\frac{4}{5} & 0 & 0 & \frac{1}{5} \\ 0 & 0 & 1 & -5 & -5 & 1 \end{array} \right) \stackrel{\frac{4}{5} R_3 \rightarrow R_2}{\sim} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 18 & 19 & -4 \\ 0 & 1 & 0 & -4 & -4 & 1 \\ 0 & 0 & 1 & -5 & -5 & 1 \end{array} \right)$$

$$\text{inv } A = \begin{pmatrix} 18 & 19 & -4 \\ -4 & -4 & 1 \\ -5 & -5 & 1 \end{pmatrix}$$

b.

$$B = \begin{pmatrix} 1 & 3 & 3 \\ -1 & -4 & -4 \\ -1 & 3 & 2 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ -1 & -4 & -4 & 0 & 1 & 0 \\ -1 & 3 & 2 & 0 & 0 & 1 \end{array} \right) \stackrel{1 R_1}{\sim} \left(\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ -1 & -4 & -4 & 0 & 1 & 0 \\ -1 & 3 & 2 & 0 & 0 & 1 \end{array} \right) \stackrel{1 R_1 \rightarrow R_2}{\sim} \left(\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & -1 & -1 & 1 & 1 & 0 \\ -1 & 3 & 2 & 0 & 0 & 1 \end{array} \right) \stackrel{1 R_1 \rightarrow R_3}{\sim}$$

$$\left(\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & -1 & -1 & 1 & 1 & 0 \\ 0 & 6 & 5 & 1 & 0 & 1 \end{array} \right) \stackrel{\frac{1}{6} R_3 \leftrightarrow R_2}{\sim} \left(\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & 1 & \frac{5}{6} & \frac{1}{6} & 0 & \frac{1}{6} \\ 0 & -1 & -1 & 1 & 1 & 0 \end{array} \right) \stackrel{1 R_2 \rightarrow R_3}{\sim} \left(\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & 1 & \frac{5}{6} & \frac{1}{6} & 0 & \frac{1}{6} \\ 0 & 0 & -\frac{1}{6} & 1\frac{1}{6} & 1 & \frac{1}{6} \end{array} \right) \stackrel{-3 R_2 \rightarrow R_1}{\sim}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{6} & \frac{1}{6} & 0 & \frac{1}{6} \\ 0 & 0 & -\frac{1}{6} & 1\frac{1}{6} & 1 & \frac{1}{6} \end{array} \right) \stackrel{-6 R_3}{\sim} \left(\begin{array}{ccc|ccc} 1 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{6} & \frac{1}{6} & 0 & \frac{1}{6} \\ 0 & 0 & 1 & -7 & -6 & -1 \end{array} \right) \stackrel{-\frac{1}{2} R_3 \rightarrow R_1}{\sim} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 4 & 3 & 0 \\ 0 & 1 & \frac{5}{6} & \frac{1}{6} & 0 & \frac{1}{6} \\ 0 & 0 & 1 & -7 & -6 & -1 \end{array} \right) \stackrel{-\frac{5}{6} R_3 \rightarrow R_2}{\sim}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 4 & 3 & 0 \\ 0 & 1 & 0 & 6 & 5 & 1 \\ 0 & 0 & 1 & -7 & -6 & -1 \end{array} \right)$$

$$\text{inv } B = \begin{pmatrix} 4 & 3 & 0 \\ 6 & 5 & 1 \\ -7 & -6 & -1 \end{pmatrix}$$

e.

$$E = \begin{pmatrix} 1 & 1 & 1 \\ -3 & -4 & -1 \\ 2 & 4 & -3 \end{pmatrix}$$

$$\begin{aligned} & \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ -3 & -4 & -1 & 0 & 1 & 0 \\ 2 & 4 & -3 & 0 & 0 & 1 \end{array} \right) \xrightarrow[\sim]{-\frac{1}{3}R_2 \leftrightarrow R_1} \left(\begin{array}{ccc|ccc} 1 & 1\frac{1}{3} & \frac{1}{3} & 0 & -\frac{1}{3} & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & 4 & -3 & 0 & 0 & 1 \end{array} \right) \xrightarrow[\sim]{-1 R_1 \rightarrow R_2} \left(\begin{array}{ccc|ccc} 1 & 1\frac{1}{3} & \frac{1}{3} & 0 & -\frac{1}{3} & 0 \\ 0 & -\frac{1}{3} & \frac{2}{3} & 1 & \frac{1}{3} & 0 \\ 2 & 4 & -3 & 0 & 0 & 1 \end{array} \right) \xrightarrow[\sim]{-2 R_1 \rightarrow R_3} \\ & \left(\begin{array}{ccc|ccc} 1 & 1\frac{1}{3} & \frac{1}{3} & 0 & -\frac{1}{3} & 0 \\ 0 & -\frac{1}{3} & \frac{2}{3} & 1 & \frac{1}{3} & 0 \\ 0 & 1\frac{1}{3} & -3\frac{2}{3} & 0 & \frac{2}{3} & 1 \end{array} \right) \xrightarrow[\sim]{\frac{3}{4}R_3 \leftrightarrow R_2} \left(\begin{array}{ccc|ccc} 1 & 1\frac{1}{3} & \frac{1}{3} & 0 & -\frac{1}{3} & 0 \\ 0 & 1 & -2\frac{3}{4} & 0 & \frac{1}{2} & \frac{3}{4} \\ 0 & -\frac{1}{3} & \frac{2}{3} & 1 & \frac{1}{3} & 0 \end{array} \right) \xrightarrow[\sim]{\frac{1}{3}R_2 \rightarrow R_3} \left(\begin{array}{ccc|ccc} 1 & 1\frac{1}{3} & \frac{1}{3} & 0 & -\frac{1}{3} & 0 \\ 0 & 1 & -2\frac{3}{4} & 0 & \frac{1}{2} & \frac{3}{4} \\ 0 & 0 & -\frac{1}{4} & 1 & \frac{1}{2} & \frac{1}{4} \end{array} \right) \xrightarrow[\sim]{-1\frac{1}{3}R_2 \rightarrow R_1} \\ & \left(\begin{array}{ccc|ccc} 1 & 0 & 4 & 0 & -1 & -1 \\ 0 & 1 & -2\frac{3}{4} & 0 & \frac{1}{2} & \frac{3}{4} \\ 0 & 0 & -\frac{1}{4} & 1 & \frac{1}{2} & \frac{1}{4} \end{array} \right) \xrightarrow[\sim]{-4 R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 4 & 0 & -1 & -1 \\ 0 & 1 & -2\frac{3}{4} & 0 & \frac{1}{2} & \frac{3}{4} \\ 0 & 0 & 1 & -4 & -2 & -1 \end{array} \right) \xrightarrow[\sim]{-4 R_3 \rightarrow R_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 16 & 7 & 3 \\ 0 & 1 & -2\frac{3}{4} & 0 & \frac{1}{2} & \frac{3}{4} \\ 0 & 0 & 1 & -4 & -2 & -1 \end{array} \right) \xrightarrow[\sim]{2\frac{3}{4}R_3 \rightarrow R_2} \\ & \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 16 & 7 & 3 \\ 0 & 1 & 0 & -11 & -5 & -2 \\ 0 & 0 & 1 & -4 & -2 & -1 \end{array} \right) \end{aligned}$$

$$\text{inv } E = \begin{pmatrix} 16 & 7 & 3 \\ -11 & -5 & -2 \\ -4 & -2 & -1 \end{pmatrix}$$

f.

$$F = \begin{pmatrix} 1 & -2 & -2 \\ 3 & -5 & -5 \\ 1 & 0 & 1 \end{pmatrix}$$

$$\begin{aligned} & \left(\begin{array}{ccc|ccc} 1 & -2 & -2 & 1 & 0 & 0 \\ 3 & -5 & -5 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array} \right) \xrightarrow[\sim]{\frac{1}{3}R_2 \leftrightarrow R_1} \left(\begin{array}{ccc|ccc} 1 & -1\frac{2}{3} & -1\frac{2}{3} & 0 & \frac{1}{3} & 0 \\ 1 & -2 & -2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array} \right) \xrightarrow[\sim]{-1 R_1 \rightarrow R_2} \left(\begin{array}{ccc|ccc} 1 & -1\frac{2}{3} & -1\frac{2}{3} & 0 & \frac{1}{3} & 0 \\ 0 & -\frac{1}{3} & -\frac{1}{3} & 1 & -\frac{1}{3} & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array} \right) \xrightarrow[\sim]{-1 R_1 \rightarrow R_3} \\ & \left(\begin{array}{ccc|ccc} 1 & -1\frac{2}{3} & -1\frac{2}{3} & 0 & \frac{1}{3} & 0 \\ 0 & -\frac{1}{3} & -\frac{1}{3} & 1 & -\frac{1}{3} & 0 \\ 0 & 1\frac{2}{3} & 2\frac{2}{3} & 0 & -\frac{1}{3} & 1 \end{array} \right) \xrightarrow[\sim]{\frac{3}{5}R_3 \leftrightarrow R_2} \left(\begin{array}{ccc|ccc} 1 & -1\frac{2}{3} & -1\frac{2}{3} & 0 & \frac{1}{3} & 0 \\ 0 & 1 & 1\frac{3}{5} & 0 & -\frac{1}{5} & \frac{3}{5} \\ 0 & -\frac{1}{3} & -\frac{1}{3} & 1 & -\frac{1}{3} & 0 \end{array} \right) \xrightarrow[\sim]{\frac{1}{3}R_2 \rightarrow R_3} \left(\begin{array}{ccc|ccc} 1 & -1\frac{2}{3} & -1\frac{2}{3} & 0 & \frac{1}{3} & 0 \\ 0 & 1 & 1\frac{3}{5} & 0 & -\frac{1}{5} & \frac{3}{5} \\ 0 & 0 & \frac{1}{5} & 1 & -\frac{2}{5} & \frac{1}{5} \end{array} \right) \xrightarrow[\sim]{1\frac{2}{3}R_2 \rightarrow R_1} \\ & \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1\frac{3}{5} & 0 & -\frac{1}{5} & \frac{3}{5} \\ 0 & 0 & \frac{1}{5} & 1 & -\frac{2}{5} & \frac{1}{5} \end{array} \right) \xrightarrow[\sim]{5R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1\frac{3}{5} & 0 & -\frac{1}{5} & \frac{3}{5} \\ 0 & 0 & 1 & 5 & -2 & 1 \end{array} \right) \xrightarrow[\sim]{-1 R_3 \rightarrow R_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -5 & 2 & 0 \\ 0 & 1 & 1\frac{3}{5} & 0 & -\frac{1}{5} & \frac{3}{5} \\ 0 & 0 & 1 & 5 & -2 & 1 \end{array} \right) \xrightarrow[\sim]{-1\frac{3}{5}R_3 \rightarrow R_2} \\ & \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -5 & 2 & 0 \\ 0 & 1 & 0 & -8 & 3 & -1 \\ 0 & 0 & 1 & 5 & -2 & 1 \end{array} \right) \end{aligned}$$

$$\text{inv } F = \begin{pmatrix} -5 & 2 & 0 \\ -8 & 3 & -1 \\ 5 & -2 & 1 \end{pmatrix}$$