# Complex numbers in standard form

Recall that the standard form of complex numbers is $a + bi$, where $a, b \in \mathbb{R}$

## Exercise 1.1

Find the complex numbers in normal form corresponding to the following expressions:

a. $\left(\dfrac{1+i}{1-i}\right)^2$

b. $(1-i)(1+i)\dfrac{2}{2-i}$

c. $(-i)^{3253}$

d. $\dfrac{1-i^2+i^4-i^6+i^8-i^{10}}{1+i+i^2+i^3+i^4+i^5}$

e. $\sqrt{i}$

f. $\sqrt{-2i}$

g. $\sqrt{1+i\sqrt{3}}+\sqrt{1-i\sqrt{3}}$

## Exercise 1.2

Prove the following properties for $z, w \in \mathbb{C}$

a. $\mathcal{R}e(z) = \mathcal{I}m(iz)$
b. $\mathcal{I}m(z) = \mathcal{R}e(-iz)$
c. $\bar{z} = 2\mathcal{R}e(z) - z$
d. $|z+w|^2 + |z-w|^2 = 2(|z|^2 + |w|^2)$

## Exercise 1.3

Show that $\text{card}(\mathbb{C}) = \text{card}(\mathbb{R})$.

# Complex numbers in polar form

Recall that the polar form of complex numbers is $r(\cos\theta + i\sin\theta)$ where $r \in \mathbb{R}_+$ and $\theta \in [0, 2\pi)$.

## Exercise 2.4

Transform the following complex numbers from standard to polar form:

a. $-3 + 3i$
b. $-4\sqrt{3} - 4i$
c. $-5i$

## Exercise 2.5

Transform the following complex numbers from polar to standard form:

a. $2\left(\cos\dfrac{1}{3}\pi + i\sin\dfrac{1}{3}\pi\right)$

b. $3(\cos(-\pi) + i\sin(-\pi))$

c. $\cos\dfrac{1}{2}\pi + i\sin\dfrac{1}{2}\pi$

### Exercise 2.6

Show that if $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2\Big(\cos\theta_{2_i}\sin\theta_2\Big)$ are complex numbers in polar form, then:

$$z_1 z_2 = r_1 r_2(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2))$$

### Exercise 2.7

Compute the following products by transforming the numbers to polar form:

a. $\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)\cdot(-3+3i)\cdot\left(2\sqrt{3}+2i\right)$

b. $(1+i)\cdot(-2-2i)\cdot i$

### Exercise 2.8

Compute the following:

a. $(1+i)^{14}$

b. $(1-\cos\alpha + i\sin\alpha)^n \quad \text{for } \alpha\in[0,2\pi], n\in\mathbb{N}$

c. $z^n + \frac{1}{z^n}$ with $z + \frac{1}{z} = \sqrt{3}$

## Complex roots of unity & polynomial equations

### Exercise 3.9

Solve the following equations on $\mathbb{C}$

a. $z^2 = i$

b. $z^2 = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$

c. $z^3 + 2 - 2i = 0$

d. $z^3 + 4 - 4\sqrt{3}i = 0$

e. $z^4 = -7 + 24i$

f. $z^4 = -7 + 4\sqrt{2}i$

g. $z^8 = \sqrt{3} + i$

h. $z^7 - 2iz^4 - iz^3 - 2 = 0$

i. $z^6 + iz^3 + i - 1 = 0$

### Exercise 3.10

Find all solutions to the equation $z^5 = 2 - 2i$, rounded to three digits.